ARKIV FÖR FYSIK Band 15 nr 8

Communicated 14 January 1959 by G. Borelius and Hilding Faxén

Liquid flow in tubes I

The transition process under highly disturbed entrance flow conditions

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With 19 figures in the text

ABSTRACT

Previously reported observations on the transition process in stream bi-refringent tube flow have been repeated in part. New photographic records are presented in order to improve the illustrative material of previous reports (Lindgren 1954*a*, *b*; 1957) with respect to the characteristics of the transition process.

Some peculiar features of the stream bi-refringent effects of White Hector bentonite suspensions in tube flow have been discovered and are briefly discussed.

An explanatory discussion shows that the turbulent spots as observed in Blasius flow and the turbulent flashes as observed in tube flow do represent two stages of development of one and the same transition process. They should not be interpreted as representing corresponding manifestations of the transition process appearing in different forms in the two types of flow.

Explanation is also given to the processes of splitting of turbulent flashes within the transition region and to the 'continuous' elongation of turbulent flashes (streaks) for higher Reynolds numbers previously observed and reported.

The significance of the fact that disturbances emanating from the tube inlet have a considerably longer time of decay than eddies generated by turbulent flash bodies is discussed and appears to have a bearing on the vortex strength of the turbulent flash bodies.

A recent determination of the intermittency factor as defined for the transition process in tube flow is examined and some necessary adjustments are proposed.

Finally, an examination of the characteristic features of the turbulent flashes confirms that the turbulent eddies are produced within wall-near layers from where they diffuse into central parts of the flow where dissipation takes place. There are some characteristic mass transfer and velocity distribution patterns which appear to play an important role in the maintenance processes of the turbulent flashes.

Introduction

In 1953 and 1954 the author presented some preliminary studies on the transition process in which it was noted that transition in tube flow always seems to take place continuously as an increasing number of turbulent flashes per time unit with increasing Reynolds numbers. I also emphasized the fact that already Osborne Reynolds (1883) reported the occurrence of such turbulent flashes within the transition region in case of low disturbance level of the entrance flow while Binnie (1945, 1947) should be credited for the discovery of the intermittent course of the transition process in tube flow under highly disturbed entrance flow conditions.

At the time, I managed to confirm and extend Binnie's observations. The work, however, was continued and Professor O. H. Faxén presented some results of the extended experiments for the 5th Congress of Applied Mechanics in Brussels 1956, while the complete work was printed not until the Spring of 1957 under the title: "The transition process and other phenomena in viscous flow" (*Arkiv för Fysik, 12,* p. 1, 1957). In 1956 also J. Rotta published a paper giving a somewhat similar picture of the transition process in tube flow.

My paper of 1957 has the character of a report of discoveries rather than a systematic analysis of several phenomena in liquid flow, which in fact appear to be quite provocative in some respects. Thanks to the support and sponsorship of the *Air Research and Development Command, U.S. Air Force,* as also of the *Swedish State Council of Technical Research* it has now been possible for me to build an improved flow apparatus so as to study selected parts of the various flow details step by step.

As a first task I have undertaken to make a detailed photographic record of the characteristic development of the transition process. This task also includes a cinematographic record which will be deposited at the *U. S. Air Force, Office of Scientific Research* and the Aeronautics Division of *the Royal Institute of Technology* in Stockholm.

Experimental technique

The same stream bi-refringence technique as reported previously has been applied also in the present studies, using a 2.4 per mille White Hector bentonite suspension in distilled water. (For a closer description of the method applied and the properties of the bentonite sols I have to refer to my paper of 1957 pp. 9–29, 30–32.)

The experimental set-up shown in Fig. 1 is arranged so that the liquid passes from the supply tank *15* into the feeding pipe line *3* (made of PVC plastics hose) via the circulation pump *16*. On its way through the pipe line *3* it passes two glasswool filters *9* and *10* and it emerges into the levelling tank *12*, where its temperature is measured by a thermometer *18*. From there, parts of the liquid pass through the trumpet-shaped tank outlet *11* and enters into the experiment tube *1*, while the rest of the liquid passes the overflow edge of the levelling tank (kept within the covering container 13) and passes directly back to the supply container *15* via the overflow pipe *2* (made of PVC plastics hose). From the 5 m long experiment tube *1* the liquid enters into a calming container *14*, where its temperature is recorded by a thermometer *19*, before it enters into either of two measuring pipes which each include an orifice plate mounted into a housing *20* each of which is connected to a pressure transducer. Each of the two pressure transducers is electrically connected to a potentiometer recorder via a balancing unit so that the rate of flow is recorded within two measuring ranges of about 1:10 to each other.

The rate of flow is regulated by either of two regulation cocks *23* located some distance behind the orifice plate housing and from there the liquid again enters into the supply container *15* via either of two calibration chambers (situated within the container). They are arranged so that calibration of the flow recorder can be done

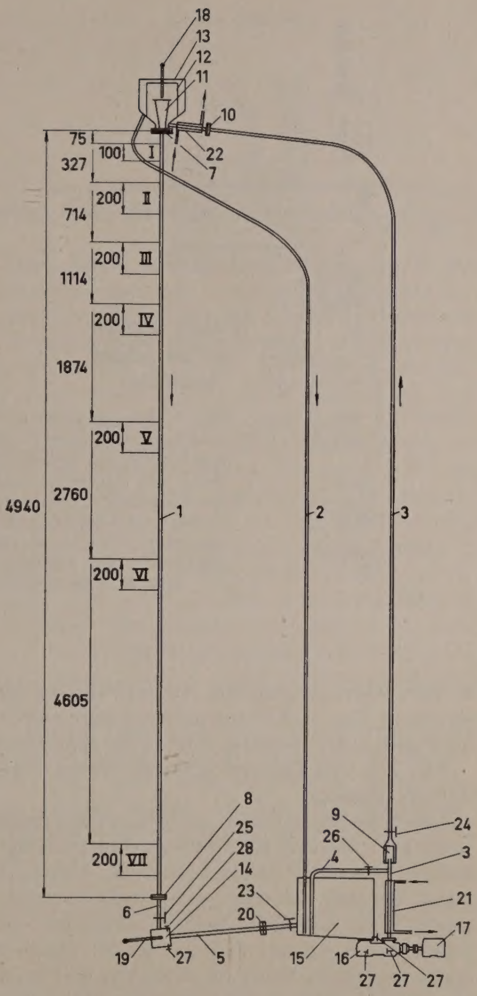

Fig. 1. Arrangement of the experimental flow apparatus

1. Experiment tube
2. Overflow return pipe (PVC plastics)
3. Feeding pipe line (PVC plastics)
4. Short circuit plexiglass pipe
5. 2 measuring pipes
6. Joint hose (PVC plastics) (see Fig. 3)
7. Inlet arrangement to the experiment tube from the levelling tank (see Fig. 2)
8. Outlet arrangement from the experiment tube (see Fig. 3)
9. Glass wool sieve
10. Glassfiber fabric sieve
11. Outlet trumpet from the levelling tank
12. Levelling tank
13. Covering tank
14. Calming tank
15. Supply container
16. Circulation pump
17. Electric motor
18, 19. Thermometers
20. 2 Housings with measuring orifice plates
21, 22. Heat exchangers
23. 2 Regulation cocks
24–26. Stop cocks
27. Tap cocks
28. Bleed cock

any time without disturbing the flow cycle or opening of the flow system which is constantly closed off from the surrounding atmosphere in order to prevent any evaporation which would otherwise change the concentration of the bentonite sols during the performance of the experiments. The total volume of liquid normally used in the flow system is 22 liters but it can vary between 20 and 25 liters, the upper limit being set by the calibration arrangement. The experiment tube *1* consists of an extruded jointless plexiglas tube with an inner diameter of 23.7 mm, the constancy of the diameter being better than 2 per cent. The total length of the tube is 4.94 m. The disturbance level of the flow entering the experiment tube is regulated by means of two parallel edges controlling the width of a slit through which the liquid has

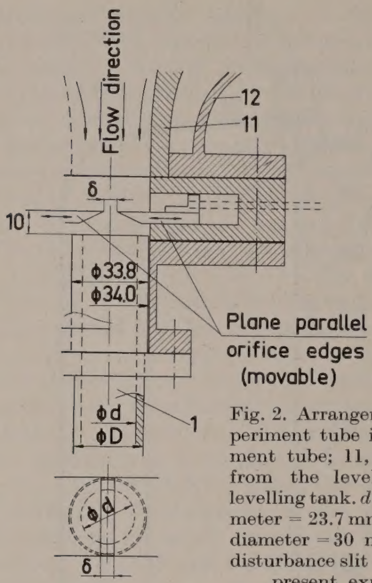

Fig. 2. Arrangement of the experiment tube inlet. 1, Experiment tube; 11, outlet trumpet from the levelling tank; 12, levelling tank. d, Inner tube diameter = 23.7 mm; D, outer tube diameter = 30 mm; δ, width of disturbance slit = 6.0 mm in the present experiments.

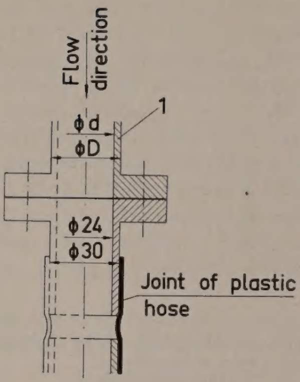

Fig. 3. Arrangement of the experiment tube outlet. 1, Experiment tube.

to pass when passing the trumpet-shaped tank outlet into the experiment tube as shown in Fig. 2. The disturbance edges are located 10.0 mm upstream of the tube inlet and in the present studies the slit was held constantly 6.0 mm wide.

Fig. 3 shows the arrangement of the outlet end of the experiment tube and is self-explanatory.

The chief construction material of the experimental apparatus is plexiglass. Much work was required for trials to construct the apparatus so that there should be no metal surfaces in contact with the flowing liquid. For instance, the inside of the pump housing has been completely covered with a film of epicote resin, and the pump rotor, originally made of stainless steel, has been substituted by one made of teflon which rotates within a stator made of synthetic rubber (neopren). The stop cocks, originally made of plexiglass and synthetic rubber, were not changed, while the interior of the regulation cocks, originally made of brass, has again been covered with epicote resin films. In addition, their interior details have been replaced by nylon replicas. The same holds for the orifice plates made of nylon and their housing made of plexiglass. The only contact between the liquid and metal surfaces in the apparatus takes place at one side of each of the two pressure transducers where a metal bellows made of bronze could not be covered by a plastics layer (here the fluid is always at rest).

The flow patterns have been recorded at seven locations or stations along the experiment tube. These stations are marked by roman numerals in Fig. 1.

The distance from the tube entry to the beginning of each observational station has been:

For station	I	75 mm,	i.e. ca.	3 tube diameters
	II	327		14
	III	714		30
	IV	1114		47
	V	1874		79
	VI	2760		116
	VII	4605		194

The length of the pictured flow fields is about 100 mm or 4 tube diameters at station I, while it is about 200 mm or 8 diameters at all the other observational stations as shown in Fig. 1.

In order to avoid lens effects of the tube, which would distort the pictures of the flow fields, the experiment tube has been equipped with rectangular plexiglass cuvettes filled with distilled water at the locations of photographic recording. However, in this way the thickness of the optically active layer of the flow not only decreases towards zero at the boundaries of the viewed flow fields but, furthermore, the difference between the refraction indexes of the water and the tube walls causes total reflection of the light within wall-near layers of the flow. The thickness of this optically inactive layer depends both on the difference of the refraction indexes and on the thickness of the tube walls relative to the inner tube radius. In the present case the tube walls are 3 mm thick, and elementary calculations indicate that the optically inactive layer should be about 2 mm thick, from within which no visual information whatsoever should be received regarding the flow texture. This state of matters is clearly confirmed by the photographs which show a dark stripe of about 2 mm thickness at the inside of each of the two tube-wall sections which in reality constitute the boundaries of the flow field. The thin light line between the inside of the tube section and the dark flow stripe is due to reflected light at the inside of the tube walls. The use of quadratic or rectangular tubes would imply a considerable improvement with regard to visual observations of flows.

The photographic imprint has been secured by means of the light from an electronic strobo-flash lamp with 2 microseconds flash duration in addition to continuous sodium light, strong enough just to reach the photographic threshold value. The strobo-flash lamp was synchronized with the camera shutter so that "instantaneous" flow patterns could be recorded. However, it was not possible to obtain a completely uniform illumination of the observational sections of the tube, as can be seen in the photographs.

The records were obtained at the seven observational stations for flows of the following Reynolds numbers:

$R=$	0	zero velocity
	130 250	purely laminar flow
	510 1020 1930	pre-transition region, disturbed entrance flow, fading turbulent patches (spots) and flashes (slugs)
	2570 2750 2870	transition region, self-maintaining turbulent slugs
	3740 7650	more or less continuous turbulent flow

The Reynolds number $R = U\, d/\nu$

is calculated on basis of the mean flow velocity U, the tube diameter d and kinematic viscosity values ν determined in my paper of 1957.

The general course of the transition process

The main features of the transition process between laminar and turbulent tube flow are demonstrated by the photographs shown in Figs. 4–11.

Fig. 4 shows the appearance of the various flow fields at zero velocity. These pictures are shown merely for comparison with the photographs where stream birefringence effects appear. (Dark flow field means zero shear stress. It is commonly assumed that the higher the local shear stress, the lighter the local appearance of the flow field.)

Fig. 5 shows the appearance of the flow texture at the various stations for a Reynolds number of $R \approx 250$. We observe that the flow appears purely laminar already from the very beginning at 75 mm (3 diameters) from the tube inlet which is the closest position to the tube inlet where the flow could be observed. This is in

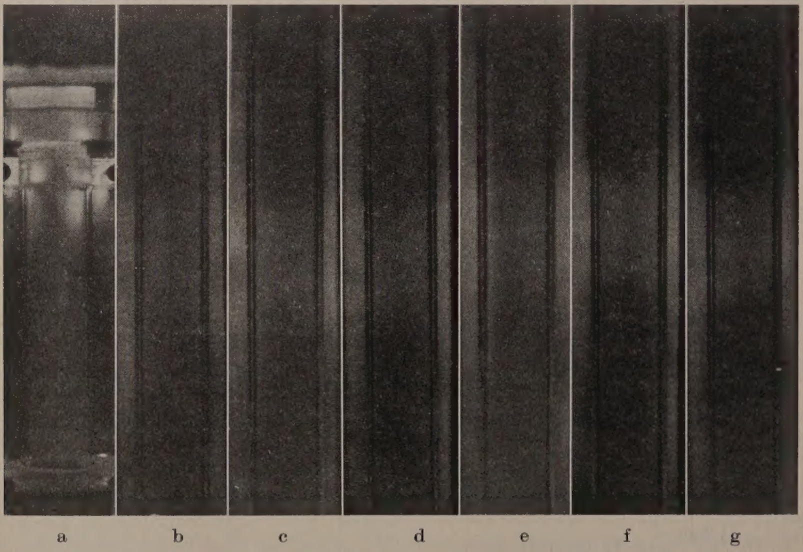

Fig. 4. The appearance of the seven pictured flow fields when the liquid is at rest, i.e. the Reynolds number $R = 0$. Note that the experimental facilities did not allow a perfectly uniform illumination along the whole length of each observational station.

a, Stn. I; *b*, Stn. II; *c*, Stn. III; *d*, Stn. IV; *e*, Stn. V; *f*, Stn. VI; *g*, Stn. VII. The location and length of each tube section are given in Fig. 1.

The flow field is in reality bounded by the light lines seen inside the outer dark stripes. The inner dark stripes are due to total reflection of light in wall-near layers as caused by differences between the refraction indexes of the water and the tube walls. Elementary calculations indicate that in the present case no perpendicular light would pass through a wall-near layer of about 2 mms thickness. Therefore, no visual information whatsoever regarding the flow texture is received from within these wall-near layers.

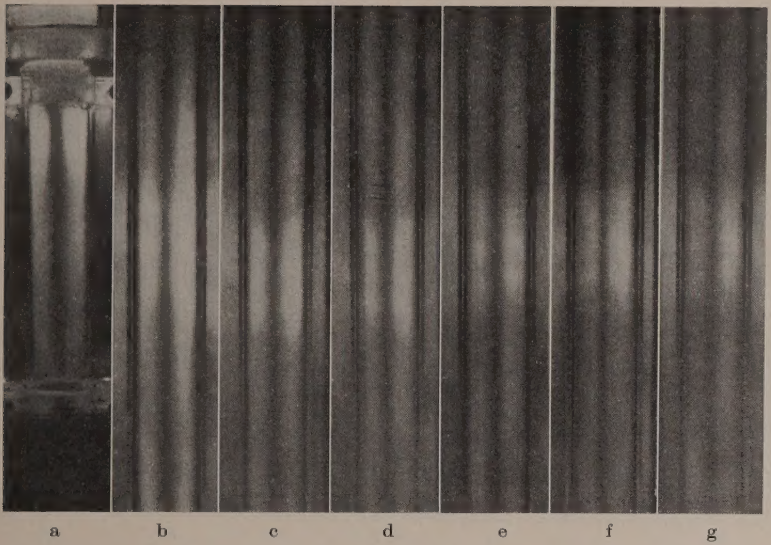

Fig. 5. $R \approx 250$. Purely laminar flow all along the tube, although the width of the disturbance slit is only $\delta = 6$ mm, because $R < R_v \approx 250$–280, the vortex limit. Note that there is a distinct development of dark veils or some sort of a structure in the flow the further downstream it is observed. This may be interpreted as due to some structural effects within the liquid which might be connected with peculiar viscosity effects reported previously (Lindgren, 1957, pp. 114–135, 137–144). Stn. I–IIV from left to right. Flow direction downwards.

full accordance with previous statements that tube flow always seems to be laminar, also very close to the inlet even under strongest possible disturbances as long as the Reynolds number does not exceed the "vortex" limit $R = R_v \approx 200$–300 (250). (See Lindgren, 1957, pp. 33 and 144, where also other references are given.)

In Fig. 6 the Reynolds number is $R \approx 1020$ and we observe a rather disturbed entrance flow which readily could be classified as being turbulent. The disturbances, however, fade away rather quickly downstream in the flow and already at Stn. II, 14–28 tube diameters, *d*, from the tube entry, there remain only some slow disturbances. These disturbances completely disappear before the flow enters Stn. III at a distance 30 *d* from the tube entry and from there the flow appears purely laminar. If we study the picture, Fig. 6*a*, we observe how the flow pattern of the rather evenly disturbed flow near the tube entry develops into some sort of gross scale eddy texture before the disturbances leave Stn. I.

Under the present circumstances the general flow pattern appears to be about the same as the above when the Reynolds number is increased to $R \approx 1930$, as shown in Fig. 7, though the small scale entrance turbulence appears more evenly distributed and reaches a little further downstream than at $R \approx 1020$. However, it appears more heavily damped than the large-scaled turbulence, as is seen from comparing the upper and lower portion of Stn. I (see picture *a*). At Stn. II (picture *b*) this leads to a sort of a distinct large-grain flow texture. The continuously disturbed entrance flow can here still be observed at Stn. IV, 47–55 *d* from the tube entry (picture *d*),

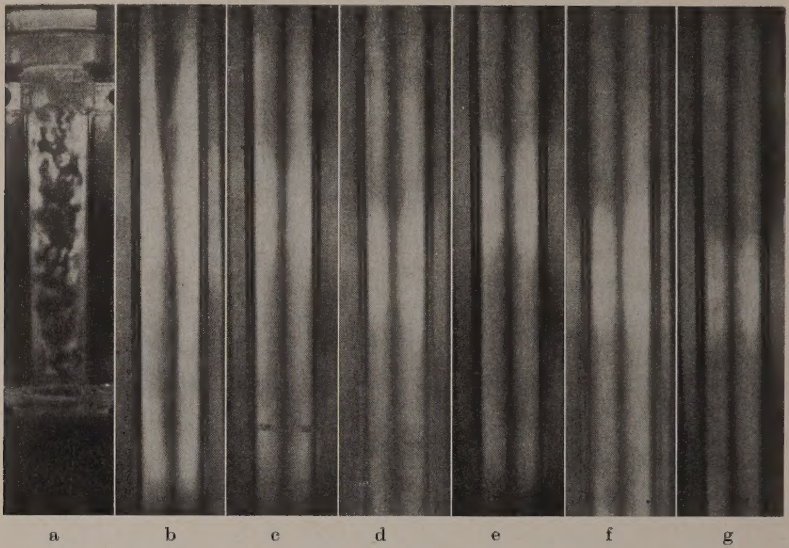

Fig. 6. Flow of the Reynolds number $R \approx 1020$. The flow appears rather turbulent close to the tube inlet in picture *a* of Stn. I. Only slow motion fading disturbances remain at Stn. II according to picture *b* while the flow appears purely laminar from Stn. III downstream (pictures *c–g*). As in Fig. 5 we note a gradual increase of the structural effects appearing in the purely laminar domain as the travelling distance increases. However, this effect is less pronounced in this figure, where $R \approx 1020$, than in Fig. 5, where $R \approx 250$.

while no disturbances ever were observed to reach Stn. V, 24 tube diameters further downstream (picture *e*), from where the flow appears purely laminar. Within the disturbed flow region the appearance of weak group disturbances was observed, evidently initiated by the continuous primary entrance disturbances. These "flashes", however, always faded away within the same entrance length of the tube as did the continuous entrance disturbances.

In Fig. 8 we have reached the real transition region at a Reynolds number of about $R \approx 2570$ where self-maintaining turbulent flashes appear as developed from the disturbances in the entrance flow. Picture *a* shows a fully developed turbulent flow pattern with some large-scale eddy formation only in the lower part of Stn. I while such structural effects are very distinctly developed at Stn. II. The disturbed entrance flow can be observed as far as to Stn. V where it appears only in the form of very slow oscillations of the black centre line. The small-structured eddies as observed in picture *b* at Stn. II fade away already before reaching Stn. III 30–38 *d* from the tube entry, although the continuously observed disturbances appear rather strong here. At the lower part in the centre of picture *c* a weak turbulent flash is observed, which does not persist for long. Picture *d* shows the disturbed entrance flow in a moment when only weak fragments of a fading turbulent flash appear at the lower portion of the flow field at Stn. IV, 47–55 *d* from the tube entry. Picture *e* shows the appearance of the body of a weak turbulent flash at Stn. V (79–87 *d* from the tube entry) which fades away before it reaches Stn. VI (116 *d* from the tube entry).

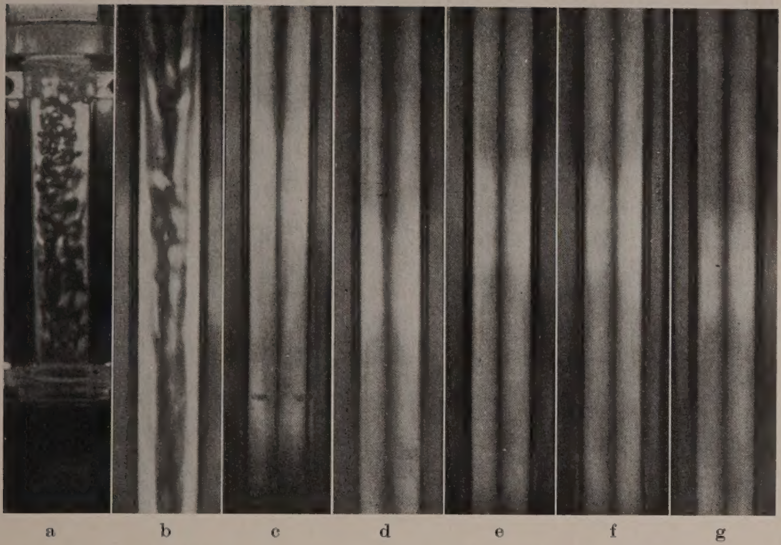

Fig. 7. In this set of photographs the Reynolds number of the flow is $R \approx 1930$. There is a fairly intense small-scaled "inlet"- or "grid"-turbulence in the flow near the tube inlet as is shown in picture *a*. These inlet disturbances develop into some kind of large "grain" formation of eddies, shown in picture *b* at Stn. II, which in some way resembles the appearance of small turbulent spots or flashes which, however, fade away rather quickly. The inlet disturbances are observable as fading slow-motion deformations as far as at Stn. IV, as can be seen from pictures *c* and *d*. In picture *c* some vague arrow-like fringes can also be observed, which could be interpreted as the last remainders of fading turbulent flashes. The inlet disturbances were never observed to reach Stn. V (picture *e*) from where the flow appears to be purely laminar. We also note less visual structural effects of the stream bi-refringent flow within the purely laminar region at the Stns. V, VI and VII for this Reynolds number than for the lower ones of Figs. 5 and 6.

At Stns. VI and VII (116–124 *d* and 194–202 *d* resp. from the tube entry) there seem to appear only self-maintaining well-developed turbulent flashes in an otherwise purely laminar flow. The pictures *f* and *g* show the appearance of the bodies of such persisting flashes. We see that there is a very intensive small-scale structure at the rear of those persisting flashes, and also that behind the main body of the slugs some kind of "veils" develop which seem to sweep the walls of the tube. If we compare the pattern of the fading turbulent flash in picture *e* with the persisting ones of pictures *f* and *g* we observe not only that no veils sweep along the walls in picture *e* but also that the turbulent fluctuations appear more gross-scaled and also that the flash is well centred around the centre line of the flow in picture *e* and does not seem to extend itself in contact with the walls, contrary to the tendencies observed on the persistent flashes in pictures *f* and *g*. This is a general observation which seems always to repeat itself. It is observed more clearly by direct visual inspection than it is shown by the photographs. A similar distinction can be observed when comparing the texture of the fading entrance disturbances at Stn. II with those of the turbulent flashes at Stns. VI and VII. Evidently the eddy distribution of the fading disturbances within the entrance flow at Stn. II (see picture *b*) is very

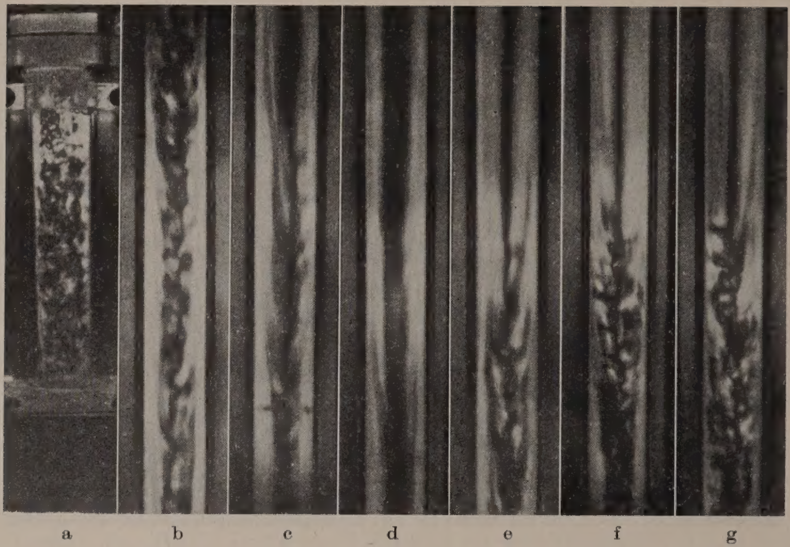

Fig. 8. Lower transition region, $R \approx 2570$. Continuous, well-developed inlet turbulence in picture *a* at Stn. I which develops to the more or less structured type of inlet turbulence of picture *b* at Stn. II while further downstream as far as to Stn. V only weak slow-motion disturbances can be observed. These disturbances, however, seem to disappear completely before reaching Stn. VI.

Within the tube sections II, III and IV the appearance of turbulent patches can now and then be observed, one of which is shown along the right wall in picture *c*, and which might fade away or develop into a turbulent flash of the weak disappearing type as shown in picture *e* or to the persisting or self-maintaining type, two bodies of which are shown in pictures *f* and *g* at Stns. VI and VII where otherwise the flow appears purely laminar.

We note that the weak fading flash of picture *e* is well centred in the middle of the flow while the persisting ones of pictures *f* and *g* seem to extend their domain in contact with the boundary tube walls.

much concentrated along the centre line of the flow while the turbulence texture is more pronounced within the body of the self-maintaining turbulent flashes shown in pictures *f* and *g* (at Stns. VI and VII), a body which extends over the entire cross-section of the tube and even appears to be in bodily contact with the bounding tube walls. Also we observe that the body of persistent turbulent flashes often exhibits an asymmetric shape while the fading flashes seem to be shaped more symmetrically around the centre of the flow.

Fig. 9 shows the appearance of the flow within the upper transition region at a Reynolds number of $R \approx 2870$ where purely laminar flow is observed only occasionally at Stns. VI and VII. Mostly there appear flashes of intensive turbulence alternating with slow-motion disturbances emanating from turbulent flashes upstream of the place of observation. The wall-near regions of the turbulent flashes move slowly along the tube walls while the eddies which have diffused from there into the more fast-moving central parts of the flow are brought forward of the main body of the flashes and might mix with the bodies of turbulent flashes located downstream of the origin of agitation during the continuous travel along the tube.

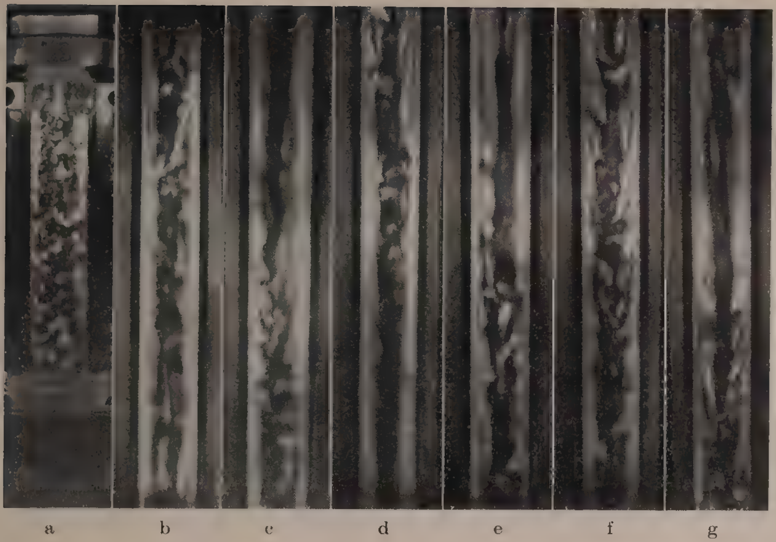

Fig. 9. Upper transition region, $R \approx 2870$. In this flow, laminar intervals appear only occasionally. A turbulent patch can be observed at Stn. III in picture *c* along the left wall which may or may not develop into a turbulent slug. A similar turbulent spot is to be noted in picture *e* of Stn. V, at the rear of a turbulent slug.

Fig. 10 shows the flow structure at a Reynolds number of $R \approx 3740$. The flow is continuously disturbed everywhere along the tube and consequently should be classified as fully developed turbulent flow. However, the records and observations definitely show that the eddy distribution is not uniform either in time or in space along the tube. The eddy distribution within the entrance flow region, i.e. the entrance turbulence (picture *a*), appears quite uniform and shows an intensive small-scale flow texture quite different from the structured "tube turbulence" further downstream. The "tube turbulence" on the other hand appears to consist of turbulent slugs (flashes) alternating with regions of fading disturbance eddies. Evidently, the turbulent slugs regenerate turbulence within their wall-near regions as they travel along the tube. The eddies diffusing into the central parts of the flow are brought forward of their generating bodies. These decaying eddies in the centre of the flow overtake other turbulence-generating bodies downstream, causing a rather complicated mixing process of various eddy distributions, inhomogeneous both in space and time.

We should observe that there appears to be no qualitative but only a quantitative difference between the flow characteristics in the "upper transition region" $R \approx 2870$ and in the "lower continuous turbulent" region $R \approx 3740$.

Fig. 11, finally, shows the flow patterns along the tube of fully developed turbulent flow at a Reynolds number of $R \approx 7650$. Also here we observe a distinct difference between the flow patterns of the entrance region (picture *a* and the first half of picutre *b*) when compared with the regions of "tube turbulence" further downstream (pictures *f* and *g* for instance). The photographs indicate a rather even distribution

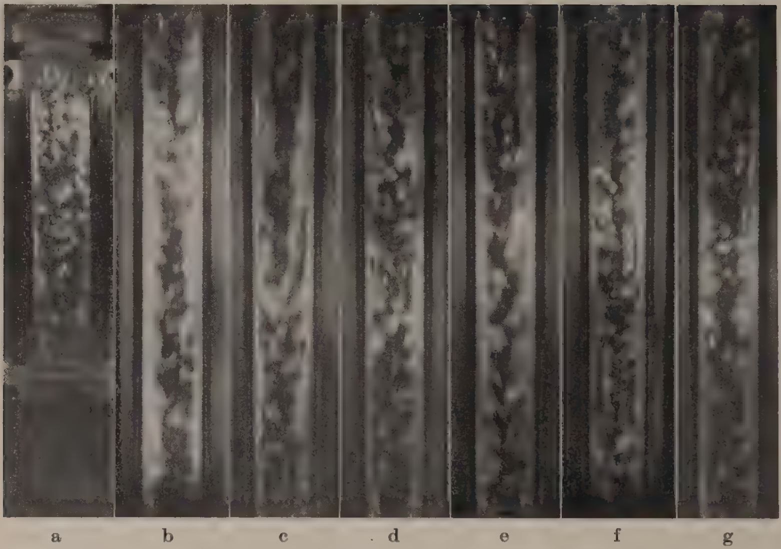

Fig. 10. Continuous turbulent flow at a Reynolds number of $R \approx 3740$. Note the development of turbulent grain texture with increasing distances from the tube inlet within the inlet length of the tube. In picture *c* at Stn. III we note a typical inhomogeneous turbulent flow pattern. Although the turbulence texture is more gross-scaled downstream of the inlet length, it appears more homogeneous than within it except for regions very close to the tube inlet where there is a region of evenly distributed small-scaled "grid" turbulence (see picture *a*).

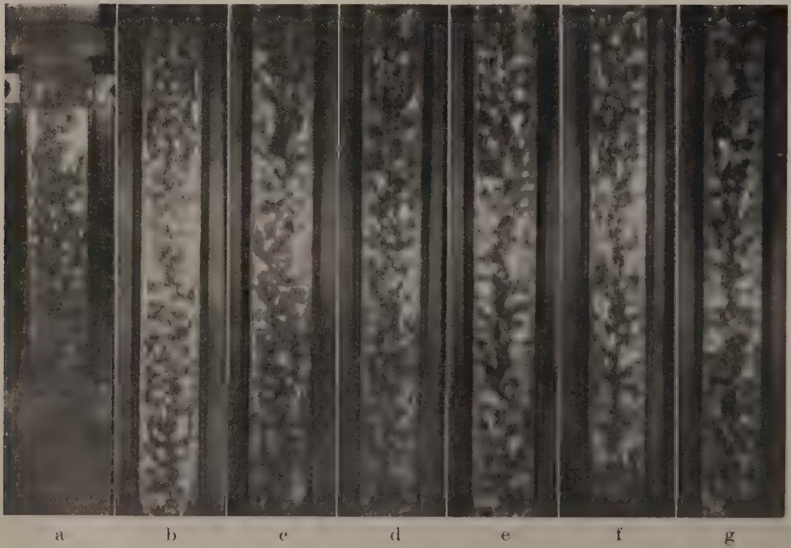

Fig. 11. Continuous turbulent flow at a Reynolds number of $R \approx 7650$, We note the distinct difference of the scale and structure of the turbulent flow patterns when pictures *a* and *b* of flow at Stns. I and II close to the tube inlet are compared with pictures *f* and *g* of flow at Stns. VI and VII farther downstream.

of small-scaled intensive turbulent fluctuations over the entire tube cross-section near the tube inlet while further downstream the eddy distribution appears more "textured" with larger eddies in the central parts of the flow and smaller eddies within the wall-near regions.

In order to record more details of the characteristics of the flow patterns under the various flow conditions described, close-up photographs also have been recorded. Of those, Figs. 12–15 show some records taken at Stns. I, II, IV and VII of flow of the Reynolds numbers $R \approx 1020$, 2570, 3470 and 7650.

Various portions of an average turbulent flash appearing at Stn. VII in a flow of the Reynolds number $R \approx 2570$ are shown in the close-up photographs of Fig. 16 while Fig. 17 shows a sequence of a cinematographic exposure of the passage of a typical turbulent flash. The time interval between each exposure is one third of a second, so the whole sequence of 13 pictures in Fig. 17 represents a total time of about 4 seconds.

Some characteristic flow features

So far, the present paper only contributes some improved illustrative material to the description of the course of the transition process published previously (Lindgren, 1954*a*; 1954*b*; 1957, pp. 33–40, 53–54, 144–154; Faxén, 1956, 1958). There are, however, some special features of the flow characteristics which deserve a closer examination. The fact that the first self-maintaining turbulent flashes in the present experiments appear first at a lower transition Reynolds number of about $R \quad \underline{R} \approx 2400$–$2500$, while previously this limit was found to be $1900 \leqslant \underline{R} \leqslant 2100$, probably is due to the rather high bentonite concentration in the experiments and is of no consequence in the present context since the course of the transition process does not visually deviate from observations reported previously.

Some effects appearing within the laminar flow region

The development of the stream bi-refringence effects along the tube at the Reynolds number $R \approx 250$ as shown in Fig. 5 is very striking. There seems to be a distinct decrease of the brightness of the flow fields as we proceed downstream. The decrease in brightness appears in the form of "structural" effects which could be described as a development of dark "veils" and "stripes" within the fluid as it moves along the tube. (At the same time it may be observed how the black centre line broadens and also might begin to perform small worm-like, slow-motion oscillations, while at the same time small fringes develop along its borders.) The development of such structural effects can also be observed if we compare the close-up photographs 14*a* and 15*a* of laminar flow of $R \approx 1020$ at the Stns. IV and VII. Evidently, this peculiar effect increases with the distance travelled by the fluid while it decreases with increasing Reynolds numbers until transition occurs. The latter statement is confirmed by comparison of the pictures of laminar flow at corresponding stations for increasing Reynolds numbers $R \approx 250$, 1020, 1930, as shown by Figs. 5, 6 and 7.

The observations reported here do not appear to be accidental but rather to be a regularly occurring phenomenon repeating itself whenever the flow was studied. Its characteristic phases of development might depend in some measure on the temperature of the liquid, and after the starting of the flow there was always some time delay before the structural effects developed.

The above phenomenon was not observed in my previous experiments. This may

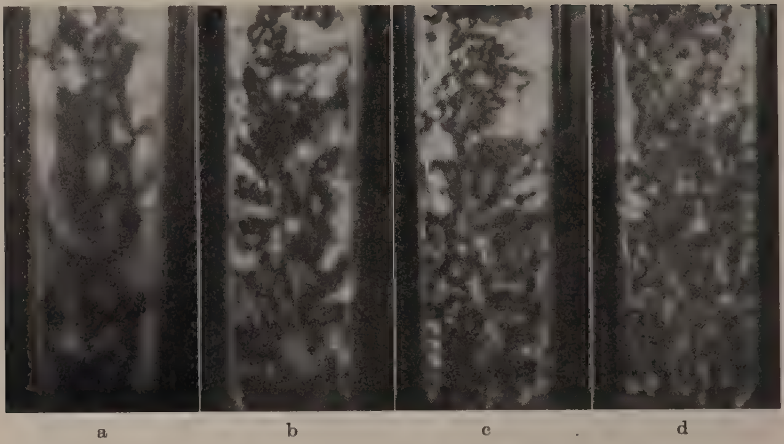

Fig. 12. Close-up photographs of the flow texture at Stn. I, 4–7 diameters *d* from the tube inlet for various Reynolds numbers: *a*, $R \approx 1020$; *b*, $R \approx 2570$; *c*, $R \approx 3740$; *d*, $R \approx 7650$. (Figs. 12–15 should be compared with each other and with corresponding pictures of Figs. 6, 8, 10 and 11 for a more complete description of the penomenon.)

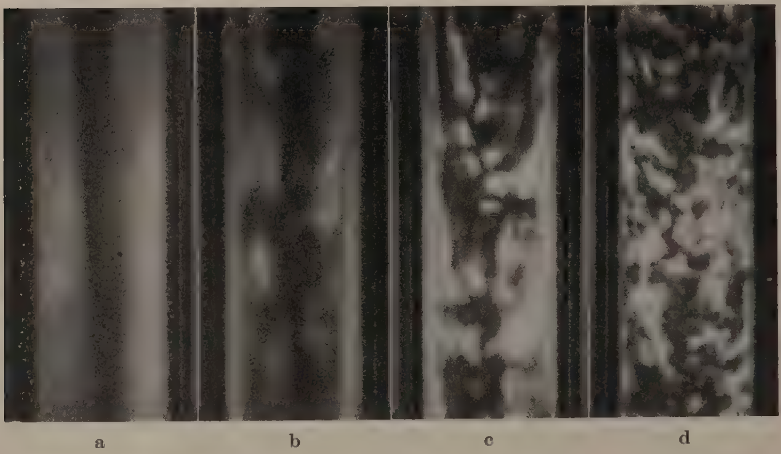

Fig. 13. Close-up photographs of the same flows as in Fig. 12 but at Stn. II, 17–20 *d* from the tube inlet.

be explained by the fact that in the first set of experiments the tubes were rather short and they were not equipped with lens-neutralizing cuvettes, nor did the apparatus allow any undisturbed study of the flow at length, while in the second set of experiments the tubes in themselves possessed polarizing properties which prevented the observation of these effects.

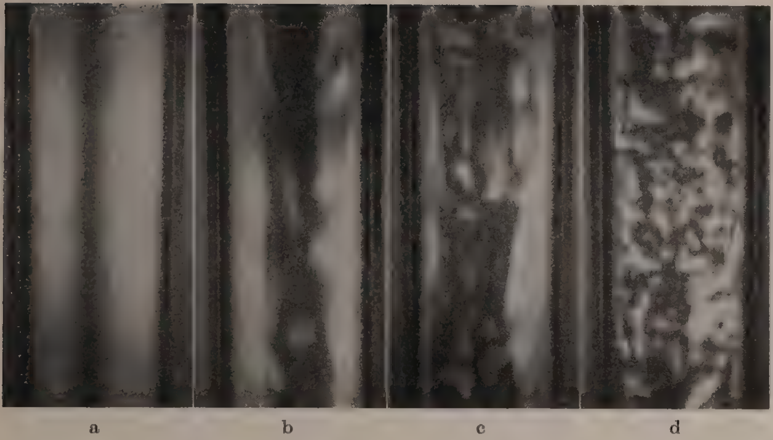

Fig. 14. Same as in Figs. 12 and 13, but at Stn. IV 50–53 *d* from the tube inlet.

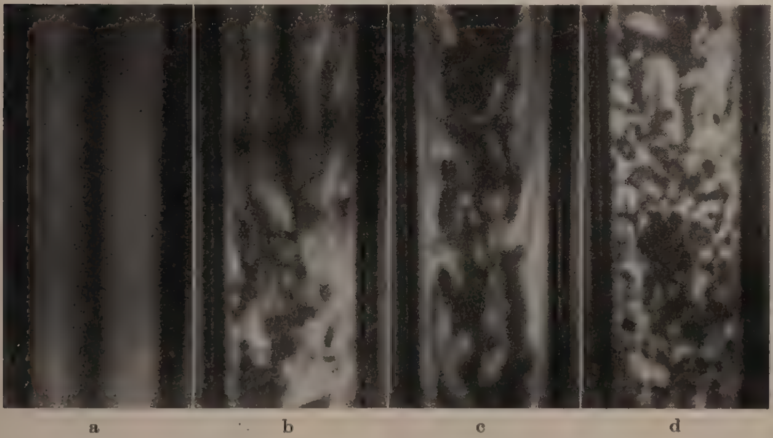

Fig. 15. Same as in Figs. 12, 13 and 14, but at Stn. VII 197–200 *d* from the tube inlet.

Of course, these structural effects might be caused only by the presence of the bentonite in the water but it does not seem unlikely that they could be connected with the peculiar viscosity effects observed and reported previously on flow of distilled water (Lindgren, 1957 pp. 114–135, 137–144), where viscosity values at low Reynolds numbers were determined to be about 20 % lower than corresponding standard values. (It should be noted that also Schnell, 1956, has reported some similar observations.)

As yet no systematic investigations have been made on the above phenomenon. It will be the subject of a later study.

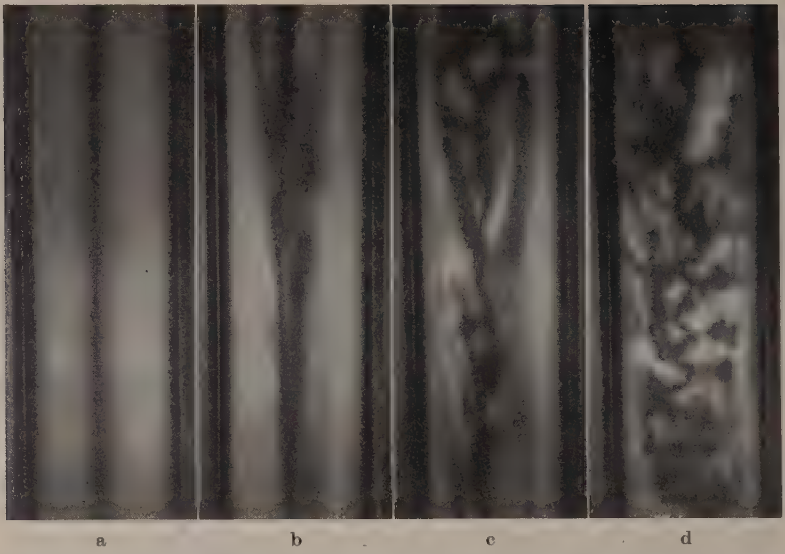

Fig. 16. Close-up photographs of various phases of the flow pattern at Stn. VII for the Reynolds number $R \approx 2570$ during the passage of a turbulent flash. *a*, Purely laminar flow; *b*, the front, *c*, the middle part, and *d*, the rear part or the turbulence maintaining body of a turbulent slug.

Turbulent spots and turbulent flashes (slugs)

In an excellent review of the present stage of knowledge regarding the transition process, Morkovin (1958) conjectures that the turbulent flashes in tube flow as observed by Reynolds (1883), Binnie (1945, 1947), Lindgren (1954*a*, 1954*b*, 1957) and Rotta (1956)—Morkovin uses the more adequate name "turbulent slug" instead of "turbulent flash"—have their counterpart in the turbulent spots as observed in flow over flat plates by Emmons (1951), Mitchner (1954) and Schubauer & Klebanoff (1955). I would like to complete Dr. Morkovin's review by stating that the turbulent spots and the turbulent slugs should not be compared with each other as implying corresponding manifestations of the transition process in two different types of flow. Instead, it is quite certain that they represent two different stages of development of one and the same transition process. The turbulent spots seem to be triggered off by the breakdown of some entrance disturbances (compare Fig. 18*a*). During the downstream transport the turbulent spots might expand and join, forming larger "patches" of turbulence. By the action of turbulent diffusion of the continuously regenerated turbulent fluctuations at the tube walls the patches finally may maintain turbulent fluctuations over the entire tube cross-section so that it appears in the form of a turbulent flash or slug.

In the present experiments the tube dimensions and the arrangements did not allow the initial stages of the development of turbulent spots to be observed. However, in Fig. 8*c* we can see a later stage of the development of a turbulent spot along the right tube wall, a little above the middle of the tube length pictured at Stn. III

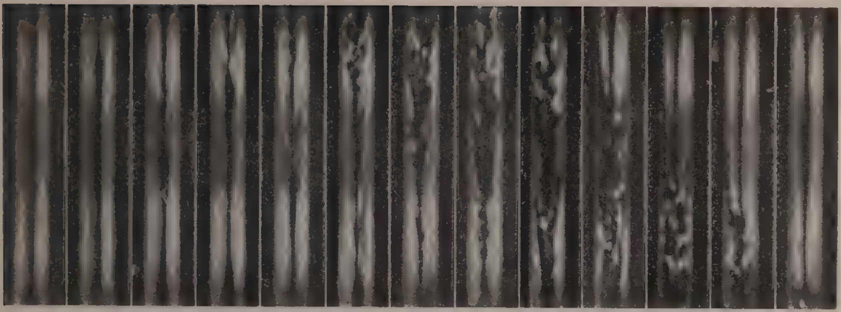

Fig. 17. The passage of a turbulent flash or slug at Stn. VII for a Reynolds number of $R \approx 2570$ (see also Fig. 16) The time interval between each exposure is $\frac{1}{3}$ second, so the complete sequence of pictures from left to right represents a time interval of 4.0 seconds. The mean flow velocity is about 0.12 m/s.

($R \approx 2570$). In Fig. 9*c* and *e* we can observe a similar stage of development of turbulent spots at Stn. III (about the middle, along the left tube wall) and at Stn. V (a little above the middle, along the right tube wall) for a Reynolds number of $R \approx 2870$.

The formation of one or several turbulent spots does not mean that they will necessarily form a turbulent slug later on. They may fade away before this stage of development is reached, just as a turbulent slug, once developed, might disappear.

The formation of spots does take place not only as a breakdown caused by primary inlet disturbances but may just as well be initiated by eddies which diffuse from wall-near parts of selfmaintaining turbulent slugs into more central parts of the flow where the transport velocity is higher. These eddies, which in themselves always are damped, consequently are brought forward of their generating bodies and may cause the formation of new turbulent spots (flashes) forward of their parent slug prior to their fading away. This scheme of development of the flow actually appears to be a most common process increasing its importance with increasing Reynolds numbers. It gives a full explanation to the splitting process of turbulent flashes within the lower transition region as it also explains the "continuous" elongation of turbulent flashes and streaks appearing for higher Reynolds numbers, both which phenomena have been observed and reported previously (Lindgren, 1957 pp. 89, 90–93, 100–102, 147–148).

The flow characteristics described here certainly have a very important bearing also on the mechanism of the fully developed turbulent flow, which subject, however, will be left aside for a later study.

The often observed asymmetric shape of the body of the turbulent slugs suggests that the body of an apparently fully developed turbulent slug in many cases actually could be interpreted as being a turbulent patch travelling along the wall and that from this patch turbulence diffuses into the flow so that the turbulent fluctuations completely fill up the entire tube cross-section. Fig. 18*b* might serve the purpose to demonstrate how a turbulent patch along the left-hand wall evidently maintains an asymmetrically shaped turbulent slug at Stn. IV (47–55 tube diameters from the tube inlet) at a Reynolds number of $R \approx 2750$.

A further demonstration of the appearance of turbulent patches is given in Fig. 19

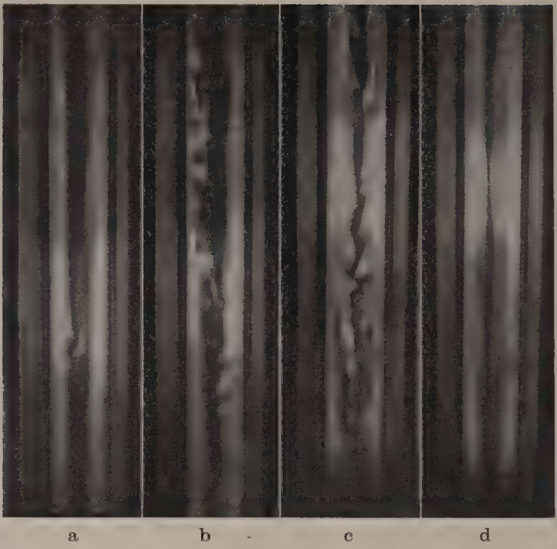

Fig. 18. Stn. IV. In picture *a* we see a phase of the process of breakdown of primarily disturbed flow at a Reynolds number of $R \approx 2570$ which may cause the development into a turbulent spot.

b. The Reynolds number is here $R \approx 2750$ and we note the presence of a turbulent spot along the upper left-hand wall. This spot seems to be in a stage of development towards a turbulent flash or slug. It already maintains the characteristic pointed front of a fully developed turbulent flash.

c. In this picture we note the typical appearance of a turbulent flash of the weak type which does not maintain itself, although the Reynolds number is as high as $R \approx 2870$ (compare picture 8*e*).

d. Here we observe an almost laminar interval in the same flow as shown in picture *c*.

($R \approx 2870$) where picture *a* (Stn. VII) shows a spot in a later stage of development into a slug, while in Fig. 19*c* (Stn. VI) we observe a turbulent patch along the left-hand wall within a turbulent slug. Fig. 19*b* (Stn. VII) shows a turbulent patch along the left-hand wall in the lower part of the picture. This patch appears to be in the first phases of development.

It is reported by Schubauer & Klebanoff (1955, pp. 10–11) that the propagation velocity of the leading edge of the turbulent spots is about equal to the local laminar flow velocity, while, according to my own measurements (Lindgren, 1957, pp. 85–92, 102, 147–148) the propagation velocity of the leading edge of the slugs (always situated in the centre of the flow) seems to be much smaller than the local laminar flow velocity (velocity along the centre line). This, however, does not imply any contradiction between the observations if we remember the above interpretation of the turbulent slugs as actually being maintained by wall-close turbulent patches, and that the trailing edge of the slugs (at the tube surface) always has a finite transport velocity, while the laminar flow velocity necessarily is zero there. Accordingly, there must be a layer some distance from the tube wall in which the transport velocity of the turbulent fluctuations is equal to the local laminar flow velocity, in full agreement with Schubauer & Klebanoff's observations.

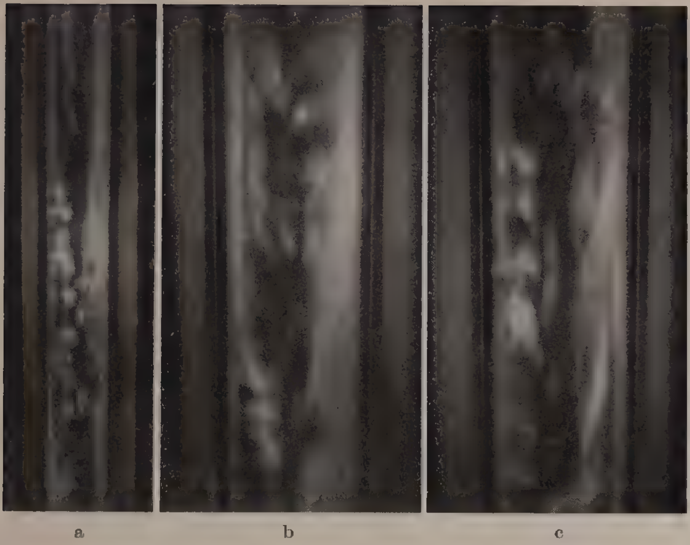

Fig. 19. Flows at the Reynolds number $R \approx 2870$. Picture *a* shows a turbulent spot at the rear of a turbulent slug passing Stn. VII (along the middle of the left wall). We note that the body of the preceding turbulent flash is quite asymmetric. It is evidently maintained by the turbulent patch along the left-hand tube wall. In picture *b* we observe a similar turbulent patch at the lower left wall in a close-up photograph of the same flow as in picture *a*. Picture *c* shows the appearance of a well-dveloped turbulent patch along the left side of the tube at Stn. VI. From this patch turbulent fluctuations diffuse into central parts of the flow and are transported forward of the patch body.

The pictures of Fig. 19 are an illustration of the inhomogeneity of the tube turbulence. Asymmetric turbulence-generating bodies do not seem to be an exception but are possibly the rule. Turbulent slug bodies which appear symmetric in recorded views might appear asymmetric from other view directions.

Evidently, measurements on the boundary layers in Blasius flow performed in wind tunnels hitherto have not allowed observations on the development of turbulent spots into fully grown turbulent slugs. The channel lengths available seem to have been only a fraction of the necessary inlet lengths. In fact, the studies on the formation and growths of turbulent spots in wind tunnel experiments have been performed under such circumstances that continuously initiated artificial spots have joined, forming continuous turbulent boundary layers, while the rest of the flow core has been laminar or of the disturbed type only. On such conditions it seems logical that there should be a smooth gradual change between the inlet flow and the fully developed turbulent channel flow downstream of the location where the disturbances, diffusing from the self-maintaining turbulent boundary layer, begin to cover the whole flow cross-section. Whether this conclusion is correct cannot be taken for certain and could be the subject for an interesting experimental study in a sufficiently long wind tunnel.

In any case, it is quite clear that spontaneously initiated turbulent spots do not cause such a continuous development of the entrance flow into fully developed turbulent tube flow. Classical experiments due to Reynolds 1883 and others, as also experiments performed recently by Professor M. R. Carstens (1957) at the Georgia Institute of Technology, have shown this fact beyond any doubt. Carstens performed his experiments on tube flow of water during the inception period of the development of the laminar boundary layers. He observed that turbulent spots were released intermittently by tube joints and roughness elements which, evidently, had been transformed into disturbance generators by inlet flow disturbances. (I have previously misinterpreted a formulation in Carstens' paper according to which the transport velocity of the front of the turbulent regions should be equal to that of the centre-line velocity of the laminar flow. This is true in Carstens' experiments where the velocity profile is square over the entire cross-section of the flow except for thin boundary layers along the tube walls. It is, however, not correct to interpret Carstens' formulation as a general statement. (See Lindgren, 1957, p. 153, last paragraph).

A direct correspondence between the formation of turbulent spots in flow over plates and the occurrence of turbulent slugs in tube flow is presented by Professor Emmons' discovery in 1951 of the turbulent spots. In his study the turbulent spots actually are fully developed, completely filling up whole lateral sections of the flow between its boundaries (wall and free surface) in the same way as in tube flow. In the boundary layer experiments in wind tunnels on the contrary, the spots are free to diffuse into central, more undisturbed parts of the flow. This may give a natural explanation to the different geometrical shape of the spots in Emmons' water layer experiments as compared for instance with Schubauer & Klebanoff's boundary layer study on flow of air in 1955.

In this connection it is interesting to note that Professor A. H. Einstein of California University has observed the appearance of fully developed turbulent slugs in open-channel flow of water, the cross-dimensions of the flow being some square feet.[1] Regrettably enough, no information could be received about the actual Reynolds number of that flow.

Some observations on the decay of eddies

A crude estimate of the diffusion of the largest inlet vortices indicates that they should disappear after about a distance of about 2.0 meters from the tube inlet at a Reynolds number of $R \approx 2570$. This estimation agrees fairly well with the observations made. On the other hand the disturbances in the flow core, continuously generated within the bodies (or spots) of stationary turbulent slugs, fade away within a distance of less than 0.5 m in front of the flash body at the same Reynolds number. Provided that the local flow velocity determines their rotational velocity, the size of the largest eddies generated by the flash bodies can be estimated to be only about $\frac{4}{3}$ of the size of the inlet disturbance eddies. Under these circumstances and if we assume that the slug body taking part in the turbulence-generating process is of about the same cross-dimensions as the largest fading eddies and that this body may be classified as some kind of a deformated toroidal vortex rolling along the tube, we find that the transport velocity of the slug at the Reynolds number in question should be about 0.94 U, where U = the mean flow velocity. This result agrees fairly well with previously reported observations (Lindgren, 1957, p. 92) according to which the transport velocity of the turbulent slug bodies decreases from about 1.0 U to 0.9 U with increasing Reynolds numbers within the lower transition region.

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[1] Personal information from Professor Einstein.

We remember that a whole spectrum of eddies of various size constitutes the turbulence-generating slug body from which disturbances diffuse into the flow core so that the whole cross-section of the middle parts of a slug is filled with turbulence. However, it is only the eddies within the flow core outside of the turbulence generating body which are damped and fade away within the rather short distance ahead of this body.

Evidently the generation of entrance disturbances implies a separation process where the size and vortex strength of the largest eddies is determined by the tube dimensions and the shape of the tube inlet as would be expected. The disturbances of the turbulent slugs, however, seem to imply a continuous regeneration of small eddies along the tube wall, which through diffusion processes constitute some sort of a vortex body, its resultant vorticity, however, being weaker than a corresponding regular torus vortex filling up the tube cross-section. If we further assume that the velocity distribution within the body of the turbulent flashes deviates from the parabolic profile towards the turbulent velocity profile, which is actually the case as shown by Rotta (1956), we note the interesting fact that there must be velocity components of the flow which are directed obliquely outwards from the centre line of the flow somewhere at the rear of the turbulent slugs where laminar flow enters into the turbulent body and becomes turbulent, while somewhere at the front of the turbulent slugs, where turbulent flow becomes laminar, mean flow velocity components must be directed obliquely inwards from the tube walls (Faxén, 1958). Evidently, these velocity components at the rear and front of the flashes are directed against the natural direction of the circulation of the flashes and would tend to weaken their vortex strength. This would imply a logical explanation of the difference in time of decay for the primary disturbance vortices as compared with those emanating from the bodies of turbulent slugs. This may also have a bearing on the maintenance processes of the turbulence production within the flashes.

The intermittency factor and the shape of the turbulent flashes

Rotta in 1956 introduced the intermittency factor γ for the transition process in tube flow. It is defined as the quotient between the time during which the flow appears turbulent and the total time of observation. Consequently $\gamma = 1$ means continuous turbulent flow, while $\gamma = 0$ should imply purely laminar flow. In Rotta's paper (p. 277) a diagram is given where γ is plotted as function of the dimensionless distance x/d (d = tube diameter) from the tube inlet. (That diagram is also reproduced in H. Schlichting's Grenzschicht-Theorie, Karlsruhe, 3rd ed. 1958, p. 349). In Rotta's experiments the point of observation nearest to the tube inlet was located 71 diameters from it. According to Rotta, γ increases with increasing distances from the tube inlet. For distances shorter than 71 d an extrapolation has been performed so that the curves smoothly deviate towards zero at the very tube inlet. I do not believe that this extrapolation is justified and correct, because if we consider the shape of the tube inlets in Rotta's experiments, it is quite clear that there must have been a rather well-developed "inlet turbulence" both at the tube inlet and some finite distance downstream of it within the flow region investigated. Consequently, $\gamma = 1$ and should not tend towards zero at the inlet end of the tube. On the other hand, if we consider only the "tube turbulence" (i.e. in this case the turbulent slugs) and separate the "inlet turbulence" from it (as evidently correctly done by Rotta), γ certainly should be zero not at the tube inlet but some finite distance, say at least 15–20 tube diameters, downstream of it. Furthermore, it is unlikely that this zero point should be one and the same for flows of different Reynolds numbers even for the same shape of tube inlet.

So far as can be concluded from Rotta's paper it is possible that his hot-wire equipment has not had a high enough response to large distorted eddies (i.e. very slow motion oscillations) which must have been present 70 tube diameters from the tube inlet at least for the experiments within the upper transition region $2400 < R < 2600$. (Rotta has shown no oscillograph record from this location along the tube.) It is, however, quite believable that the slow disturbances would be indicated so vaguely by the hot-wire device that no definite conclusions could be drawn from the records in this respect.

It is interesting to note that even the few hot-wire records shown in Fig. 8 (p. 265) in Rotta's paper suggest that the front must be pointed while the rear should be somewhat square. According to those records the local laminar flow velocity is higher than corresponding local velocity of the turbulent flow within the slugs in the central parts of the flow, while nearer the walls the oscillograms show that the local mean flow velocity within the turbulent slugs is higher than the corresponding laminar flow velocity. The local turbulent average flow velocity appears to be about equal to the corresponding laminar flow velocity at a distance of about $0.3\ d$ from the tube wall. Further, the records indicate a rather slow change of the velocity from the laminar to turbulent regions at the slug front in the centre of the tube while nearer to the tube walls this change of velocity appears more abrupt. On the other hand, the change of local average flow velocity between the laminar flow and the turbulent flow at the rear of the turbulent slugs takes place rather steeply both in the central parts of the flow and in the wall-near regions where no characteristic differences can be observed between the velocity change at the rear or at the front of the slugs. Another feature of Rotta's records is that, on an average, γ appears to decrease as the tube wall is approached. All these characteristics of Rotta's hot-wire records seem to indicate that the turbulent flashes or slugs should have a pointed front and a rather square rear also in tube flow of gases in agreement with my present and previous reports on tube flow of liquids (Lindgren, 1954*a*, *b*, 1957).

It should further be noted that there is also good agreement between the quantitative observations due to Rotta and to my previous observations reported in 1957 with respect to the relative extension velocity of the turbulent slugs.

Turbulent tube flow

The statements given above about the appearance of turbulent spots and slugs would allow some interesting conclusions to be drawn about the properties of fully turbulent tube flow. However, leaving aside this subject at the present time, I merely wish to point out that wall-close turbulent patches appear to be present in the "fully developed" turbulent flows of Reynolds numbers $R \approx 3740$ and 7640 as shown in the photographs of Figs. 10, 11, 12, 13*c* and *d*; 14*b*, *c* and *d*, and 15*c* and *d*. These patches distinguish themselves from the core turbulence through their characteristic stretching along the walls.

ACKNOWLEDGEMENTS

The research reported in this paper has been made possible through the support and sponsorship of the *Air Research and Development Command, United States Air Force* through its *European Office* under Contract No. AF 61 (514)-1202 as also through the sponsorship of the *Swedish State Council of Technical Research* under Contract No. 1875.

The experiments have been performed within the administration of the Division of Mechanics I (Professor O. H. Faxén) and on laboratory premises belonging to the Division of Hydraulics Engineering (Professor Magnus Oledal).

Credit is due to Mr. John Persson for very fine mechanical work in constructing the flow apparatus, and also to Messrs. Erik Johansson and H. Elfving for having made valuable contributions thereto. I am also indebted to Professor Faxén and Dr. Sune Berndt who have made valuable comments during the preparation of this paper.

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REFERENCES

Binnie, A. M., A double-refraction method of detecting turbulence in liquids. Proc. Phys. Soc. London *57*, 390 (1945).

Binnie, A. M., and Fowler, J. S., A study by a double-refraction method of the development of turbulence in a long cylindrical tube. Proc. Roy. Soc. *A 192*, 32 (1947).

Carstens, M. R., Transition from laminar to turbulent flow in a pipe. Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Paper 1450, HY 6, December 1957.

Emmons, H. W., The laminar-turbulent transition in a boundary layer—Part I. Journ. Aeron. Sc. *18*, 490 (1951).

Faxén, O. H., The growth of turbulent flashes. IX^e Congrès International de Mécanique Appliquée, Bruxelles 1956. Tome 3, p. 435.

—— Turbulent flashes. Arkiv för Fysik *13*, 235 (1958).

Lindgren, E. Rune, Some aspects of the change between laminar and turbulent flow of liquids in cylindrical tubes. Arkiv för Fysik 7, 293 (1954*a*).

—— Note on the flow of liquids in tubes. Appl. Sci. Res. (A) *4*, 313 (1954*b*).

—— The transition process and other phenomena in viscous flow. Arkiv för Fysik *12*, 1 (1957).

Mitchner, Morton, Propagation of turbulence from an instantaneous point disturbance. Journ. Aeron. Sc. *21*, 350 (1954).

Morkovin, M. V., Transition from laminar to turbulent shear flow—A review of some recent advances in its understanding. Trans. ASME *80*, 1121 (1958).

Reynolds, Osborne, An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and the law of resistance in parallel channels. Trans. Roy. Soc. *174*, 935 (1883).

Rotta, J., Experimenteller Beitrag zur Entstehung turbulenter Strömung im Rohr. Ingenieur-Archiv *24*, 258 (1956).

Schnell, Erhard, Slippage of water over nonwettable surfaces. Journ. Appl. Physics *27*, 1149 (1956).

Schubauer, G. B., and Klebanoff, P. S., Contributions on the mechanics of boundary layer transition. Symposium on Boundary Layer Effects in Aerodynamics, 31st March–2nd April 1955, National Physical Laboratory, England.

Tryckt den 19 mars 1959

Uppsala 1959. Almqvist & Wiksells Boktryckeri AB